Introductory Lectures on Networks

(A simplified informal version of the attached published papers)

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1) **Overview of Networks and Their Significance**

a) Importance of Networks:
   i) There may be no problem more critical to our survival than the understanding of networks.
      - The research explosion concerned with network theory and applications has made us aware that the understanding of networks is essential to the understanding and optimization of the vast range of social structures, perhaps even the rise of mankind as the dominant living species.
   ii) Networks are Pervasive:
      - Networks are pervasive as: communication linkages (internet, telephone, and personal meetings), transportation systems (roadway, railway, airway, waterway), utility systems (power grids, water-sewer networks, and gas lines), all financial transactions and currency flows, all electrical/computer circuits/networks, biological networks (health, genetic, ancestral-family, neural, circulatory, disease), linguistic/anthropological/cultural structures, and a spectrum of other composite and derivative networks often involving geospatial distributions.
   iii) Thus understanding both the underlying structures of these topologies and their dynamical evolutions is of the greatest importance in understanding the social systems which they describe.
   iv) It is only with such scientific understanding of (social) networks that we can even begin to optimize them for efficiency and productivity, for social order, health, and our economy, and ultimately our very survival.
   v) There may be no more important problem than this if we can begin to more optimally address the cultural/resource and geographical boundaries that lead to war, the spread of disease and information, the optimal allocation of food and essential supplies, trade and the flow of capital, and the optimization of the access to knowledge, education, and communication among people.

b) Social Networks:
   i) One notes that essentially all networks are in essence ‘social networks’ since they would not exist without the linkages of living things.
   ii) The internet and other related forms of social networks have led to a much more intense study of networks both in their general and specific forms and has made our lack of understanding very apparent.
   iii) Progress made in any network domain is useful to our core interest here of health related networks and in turn health and disease networks are interlocked with all other networks mathematically.
   iv) It is just recently, with the use of the mathematics of networks, that scientists have begun to formulate the extremely convoluted, complex, nonlinear, and chaotic systems of social phenomena
   v) The literature on social networks specifically has truly exploded into a now vast field. Wikipedia provides an excellent introductory survey where these and other critical references can be found.

c) Comparison to the Fundamental Laws of Physics.
   i) Physics:
      - The laws of the fundamental physical sciences represent the state of any system of particles as a vector (set of ordered numbers), whether the system might be one particle, or a hundred, and whether the applicable theory is classical, relativistic, or quantum mechanical.
   ii) Networks:

    - But social systems are defined by networks which are represented by matrices (square arrays of numbers) that give the relationships among entities as the strengths of connection among all possible pairs of nodes, potentially along with attached vectors of attributes (such as demographic profiles of the nodes) associated with each node and likewise attribute vectors describing each link and connection degree.
    - These connection matrices, along with these node and link attribute vectors, act for social systems to provide core properties of the state of the system and are of exceptional mathematical, computational, and conceptual difficulty.
    - Living things form social structures in order to best survive as collectives on multiple levels to achieve the sharing (trade) of information, energy, health, currency, food, and safety from threat..
  - iii) Fundamental Linearity of Physics Laws verses Nonlinearity of Social Networks:
    - The fundamental principle of linear superposition in the physical sciences is that if we know the separate effects (e.g. the electric field from one particle and also that of a second particle) then the total effect of the two is the linear vector sum of each separately.
    - But this is totally wrong in the social sciences since the nonlinear interaction effects dominate the separate effects thereby giving an extremely nonlinear system.
    - Thus traditional reductionism cannot be utilized.
    - Also, whereas the dynamical evolution of fundamental physical systems proceeds by a linear evolution (matrix) operator acting upon the vectors of the state of the system, such transformations that might describe the evolution of the connection matrices that describe networks, and attached attribute vectors, are extremely nonlinear with chaotic components and are generally unknown.
- d) Networks are Poorly Understood
  - i) Network analysis constitutes one of the most difficult of all mathematical problems:
    - Their topologies are not classified or understood;
    - Current metrics, that reduce the vast set of values to smaller sets in the matrix of connectivity are of marginal use; and most profoundly,
    - The primary representation of a network as a matrix is ambiguous and arbitrary as the diagonal is missing (because there is no meaning to connecting a thing to itself).
  - ii) Thus to more deeply understand social systems, we need a firm unambiguous mathematical footing for the connection matrices, metrics that hierarchically describe such systems increasingly from dominant to lesser effects, and an intuitive structure that can guide the mathematics.
  - iii) Furthermore we need both macro and micro fundamental social network structures with accurate unambiguous representation and dynamical evolutions that can be analyzed both topologically and dynamically with useful intuitive metrics.
- e) The Characteristics of Networks
  - i) Every network consists of points or nodes which can be numbered 1, 2, … n and a set of non-negative numbers that connect each pair of nodes as expressed in a square matrix C_{ij} (called the connection, connectivity, or adjancy matrix) giving the strength of connection between the two nodes.
    - Most of the values of C are zero and thus this is often called a 'sparse' matrix.
    - One example might be the named airports in the US where C_{ij} represents the number of flights (or passengers) per week that pass from airport i to airport j.
    - One notices that the matrix is not necessarily symmetric and that the diagonal of the matrix is not defined as there is no meaning to connecting a thing to itself.

- So, to understand networks, we must understand square matrices, with non-negative values off the diagonal and with missing diagonal terms (since the connection of a thing to itself is not defined).

f) Types of Social Networks:
  - i) A network can first be classified by whether the connections represent a real or virtual flow. An example of a real flow would be the passengers that are flown from airport i to airport j or as another example a financial transfer from bank account i to bank account j. An example of a virtual flow might be a friendship, or a genetic relationship, where the linkage would be a weight of the strength of the connection but not measuring any real entity flow.
  - ii) A network could also be classified by whether the flows represent a conserved substance such as people flying from i to j or again a financial transfer (since the total number of people and funds are constant). A nonconserved flow might be an information network (emails, phone calls) or a disease network where one node infects another.

g) Network Problems, the Specifics:
  - i) (P1) Networks can be very large arrays, and as there is no natural order for the nodes, it follows that the same topology is described by n! different matrices (ways of numbering the nodes and thus ways of forming the C matrix) thus making it computationally impossible to even compare two networks to see if they are the same.
  - ii) (P2) Networks do not have a good ‘intuitive foundation’ in terms of the eigenvalue analysis or other expansions (other than clustering and graphical representations).
  - iii) (P3) Since the diagonal is missing, and can only be given an arbitrary value such as ‘0’, it follows that any eigenvalue/eigenvector analysis of the matrix is likewise ambiguous. Specifically this means that there is not a ‘nice well defined’ mathematical foundation for network expression other than as an ‘off-diagonal piece of a matrix’.
  - iv) (P4) There are no known ways to uniquely classify all of the possible topologies (a very formidable mathematical problem).
  - v) (P5) We lack a sense of how to ‘expand’ a network as a series of terms, polynomials, or functions that represent ‘decreasing levels of importance’ as we are able to do in other areas of mathematics using Taylor series, binomial and multipole expansions (for mass and charge distributions), Fourier series and integral expansions, and generally all of the expansions in orthogonal functions.
  - vi) (P6) The dynamics of networks is in itself orders of magnitude even more complex, chaotic, and highly nonlinear.
  - vii) (P7) We furthermore do not have the fundamental core metrics with which to track dynamical evolution. There are no invariants or constants of the motion (such as energy, mass, momentum, angular momentum, charge), which so serve in the physical sciences.
  - viii) (P8) From the practical point of view, spite of the proliferation of social networks, it is very difficult to get data and certainly accurate data because of both personal privacy issues for people and because of trade secret issues for companies. This is especially true for health data.
  - ix) (P9) Finally one might ask if a network is ‘optimal’ and generally how do we even define ‘optimal’ for different applications as well as whether a network is becoming ‘more optimal’ for some purpose over time.

2) Markov Transformations

i) Markov transformations defined: Markov 1906:

- Markov transformations are linear (matrix) transformations that when acting on a vector of non-negative values (positive or zero components) preserve the sum of that vectors components (i.e. the sum of the values is invariant) and where the new transformed vector components must also have non-negative values which also sum to unity.
- Thus if M is a Markov transformation then Y = M X will have $\Sigma y_i = \Sigma x_i$ and $x_i >= 0$ implies that $y_i >= 0$.
- You can compare this to rotations Y = R X where the radius (sum of the squares) is preserved $\Sigma y_i^2 = \Sigma x_i^2$
- Markov transformations are an important operation on probabilities, or numbers of things that are preserved but simply change categories such as rearrangements.
- A most important aspect of Markov transformations is that they do not have an inverse and can mathematically represent diffusion (red dye poured into clear water, or dirt/disorder in our houses). Thus they represent an increase of entropy.
- Because Markov transformations do not have an inverse, they were never studied from the point of view of group theory, because groups all have inverse transformations.
- It can be shown that all Markov transformations are square matrices that consist of non-negative (positive or zero) numbers where each column sums to unity (one).
- Markov theory is also related to the second law of thermodynamics concerning the gradual dissipation of free energy into heat with increased entropy as well as the degradation of transmitted or stored information.

ii) Example:

- Consider the Markov matrix (0.9, 0.1; 0.2, 0.8)
- Let it act on the money that Peter has ($18) and that Paul has ($10). X = (18,10)
- Confirm that after one transformation they have Y = (18.2, 9.8)
- We have robbed Paul to pay Peter but the total sum of money is the same ($28).
- You can verify that we gave 10% of Peters money to Paul but we also gave 20% of Paul's money to Peter.
- This process will always preserve the total amount of money (or substance or things).
- With a little thought, one can verify that all of diffusion can be represented by Markov transformations and thus they give the increasing disorder that we associate with the 'arrow' or direction of time. (A persons house never just gets more organized or cleaner on its own!).

3) Linear Transformations and Groups

a) Groups

i) A mathematical group is a set of objects (say A, B, C, …) and a multiply operation (*) that has (a) closure $A*B = C$ (another member of the set), (b) is transitive: $(A*B)*C = A*(B*C)$, (c) has an identity transformation "I" so that $I*A = A*I = A$ leaving it unchanged, and (d) has an inverse A^{-1} for every element so that $A^{-1} * A = A * A^{-1} = I$.

ii) One simple example is the set of the identity and the reflection R in a mirror. Note that $R*R = I$.

iii) Another example is the set of four rotations of a square that leave it invariant (by 0,90, 180, and 270 degrees).

b) Continuous (Lie) groups of transformations:

i) Consider the continuous transformations of rotation: $R(\theta) = (\cos\theta , \sin\theta; -\sin\theta ; \cos\theta)$

ii) There are an infinite number of these transformations (corresponding to each of the infinite values of the angle θ).

iii) About 1890 Sophius Lie invented a way to study all of these by studying an associated infinitesimal transformation, and then exponentiating it to get the original transformation.

- For rotations, if we look at an infinitesimal value of the angle we get
- $R(\varepsilon) = (1, \varepsilon ; - \varepsilon, 1) = I + \varepsilon (0, 1; - 1\ 0) = I + \varepsilon L$ where L is called the infinitesimal generator.
- Lie was able to show that $R(\theta) = e^{\theta L}$
- This means that we can study a single L rather than the infinite number of rotations.
- For rotations in three dimensions, there is a set of three Lx, Ly, and Lz for rotations about each axis and thus one only has three objects that are needed to study all rotations in three dimensions
- The resulting set of L matrices are called the Lie algebra for that Lie group.

c) A Linear Vector Space (LVS) is a set of objects called vectors (A, B, C…) that close under addition (A+B=C) and which can be multiplied by a number 'a' thus $a A = B$ thus allowing also the negative numbers and thus for vectors to be subtracted.

i) There is a minimal set of vectors (V_1, V_2, … V_n) for an 'n' dimensional space, called the basis for the space, that allows any vector to be represented as $A = a_{1\ V1} + a_2 V_2 + \ldots$

ii) With respect to this basis, one can then express the vector A as the set of numbers, (a_1, a_2, …) which are called the components of the vector with respect to that basis.

iii) By defining a product on the LVS, one can make a much richer mathematical structure. One example is a metric space and the other example is Lie algebra.

d) A metric space is a LVS with a (scalar) product defined for every two vectors using a metric g_{ij} as $A*B= \Sigma_{ij}\, g_{ij}\, a_i\, b_j$ where the tensor g_{ij} is called the metric for the space.

i) The power of this expression is because A*B also = $|A|\,|B| \cos\theta$ e thus giving the magnitude of a vector and the angle between any two vectors. This makes the space 'metric' (to measure').

ii) Note that this scalar product or dot product is always a number (real or complex)

e) A Lie algebra is a LVS with a different type of product where $[A,B] = AB – BA$ and thus is antisymmetric and gives another element of the LVS.

i) Using the basis vectors (L_1, L_2. …) for the LVS one can write $[L_i, L_j] = c_{ij}^{k} L_k$ (with an implied sum over the k).

ii) This along with another condition that $[A, [B, C]] = 0$ over cyclic permutation of the three 'vectors' defines a Lie algebra.

4) Linear Transformations Contain All Markov Transformations

- a) The general linear group of continuous transformations in n dimensions over the real numbers, GL(n,R), consists of all continuous linear transformations (that have an inverse) and is represented by an n x n (invertible) matrix of real numbers.
  - i) Such transformations include rotations, and the Lorentz transformations of the theory of relativity as well as all the unitary transformations in quantum theory.
  - ii) Transformations allow us to study symmetry such as rotational symmetry or other invariance.
  - iii) Since we wish to generate all continuous linear transformations, we will need all possible infinitesimal generating matrices which are easily listed as having a ‘1’ in the i,j position and a ‘0’ in all other positions.
  - iv) There are (as might be expected) n^2 such matrices since we can put the ‘1’ in any of the n^2 positions.
- b) Abelian Group of Scale Transformations:
  - i) Consider the generator (Lie algebra) element $(A^{ii})_{ij}$ which has a 1 at the ii position and a 0 at every other position.
  - ii) If we form $T(a) = e^{aAii}$ then this is obviously e^a at the T_{ii} position, 1 at other diagonal positions, and zeros everywhere off the diagonal.
  - iii) These transformations multiply that one axis by e^a and multiply all the rest by ‘1’ thus leaving them unchanged so it just makes that one axis longer or shorter by that factor.
  - iv) We call these scaling transformations and the group is called Abelian because every transformation commutes with all the other elements in the algebra: ($[R,S] = RS - SR = 0$).
- c) A Markov Type Group:
  - i) Now look at the off-diagonal algebra (generators) and rather than using just a ‘1’ at each off diagonal position, let us place a ‘-1’ on the corresponding diagonal of that column.
  - ii) This makes the sum of the elements in each column of the generator equal to zero.
  - iii) Formally this defines the m,n matrix element as an $(L^{ij})_{mn} = 1$ if i=m and j=n and also -1 if j=m and j=n (the diagonal term).
  - iv) There are obviously n^2-n such L matrices corresponding to every place off the diagonal that one can place a ‘1’.
  - v) For example lets take L = (0,0; 1, -1)
  - vi) One can expand the exponential and show that e^{aL}= (1, 0 ; e^{-a} , 1 – e^{-a})
  - vii) Thus we can write that the General Linear Transformation is G = A + M
  - viii) These transformation move one over a flat plane that is perpendicular to the vector (1,1,1,…1) and in two dimensions this is simply the line that goes between the points (1,0) and (0,1).
  - ix) This is in contrast to the rotations which move one over a sphere with a constant radius, but here one moves over a flat plane in n-1 dimensions.
  - x) These transformations include all the Markov transformations (we call this a ‘Markov Type’ Lie group because it preserves the sum but does not preserve the positive definiteness of the components of the vector as M can also take one to negative values of the coordinates which we must now prevent.
- d) The Markov Monoid (MM)
  - i) One can easily verify that if the parameters that multiply the L generators are all non-negative, then one never gets a transformation that takes one to negative values.
  - ii) In that process however, one gives up the inverse of the transformation and we end up with a group without an inverse which is called a ‘monoid’.
  - iii) Now with this restriction to non-negative values of the L multipliers, we always get a Markov matrix: One notices that the sum in each column is ‘1’ and that all elements are positive or zero <u>as long as a is positive</u> (which we now require).

iv) In general one can show that any linear combination of these off-diagonal matrices (using ONLY positive or zero coefficients) are Markov matrices and this is called a Markov monoid (MM).
v) Furthermore all continuous Markov matrices are generated in this manner.
vi) This result was highly significant because it tightly connected the theory of Lie groups with the theory of Markov transformations allowing the theorems and insights in one field to be utilized in the other.

5) **<u>Networks are 1-1 (Isomorphic) with Markov Monoids (MM)</u>**
   a) Every network is a MM and conversely:
      i) Recalling that any network is an off diagonal set of non-negative numbers, it now follows immediately that we can multiply the appropriate MM generator by the value in the off diagonal position for any network, and end up with a Markov monoid matrix that will generate a valid Markov process.
      ii) This is a consequence of the fact that each diagonal is defined as the negative sum of off-diagonal elements in that respective column. For example if a network is represented by a connection matrix C = (?, 2, 5; 4, ? 1; 1, 0, ?) (note that we used the ? to indicate the diagonal unknown elements), then the Lie algebra generator will be C = (-5, 2, 5; 4, -2, 1; 1, 0, -6)
      iii) Thus any network C gives exactly one MM generator for a continuous Markov transformation and conversely any MM generator defines, via its off-diagonal elements, a network with that C matrix.
      iv) This important result now connects the study of the complete topology of all networks to the study of the equivalent MM and its associated unique Markov transformation.
      v) The power of one branch of mathematics can now be used as we shall see, in another branch.
      vi) This result also has an immediate positive consequence, namely that the diagonal of the C matrix is exactly defined and is unambiguous as a MM.
      vii) This puts network theory on a firm mathematical unambiguous footing.
   b) Important practical consequences:
      i) Eigenvalue analysis is now well defined and is a critical metric for the topology
         - The first important consequence is that since the C matrix now has its diagonal determined and is unambiguous, that all eigenvector and eigenvalue analysis is well defined.
         - With some thought, one can show that the associated eigenvalues are all ‘0’ or negative with the ‘0’ value being associated with the equilibrium eigenvalue, and all the other (negative) eigenvalues being associated with flows of an associated diffusion rate that is exponentially deceasing for the corresponding eigenvector.
         - There are also cases (since the resulting C matrix may not be ‘normal’ where there can be complex eigenvalues and in this case this eigenvalue gives the angular velocity of a circular flow of conserved entity under the Markov transformation.
         - This is very analogous to the physical system of coupled harmonic oscillators.
      ii) Entropy can be defined for each column providing another set of critical metrics for the topology
         - The second equally or perhaps more important consequence is that since the resulting Markov matrix has columns that are non-negative and sum to unity, each column can be interpreted as a probability distribution.
         - Thus it follows that each column can support a well defined concept of entropy (either Shannon or Renyi’) on each column
         - This entropy value measures the order or disorder of the incoming (columns) or outgoing (rows) flows of the conserved substance such as probability to the node in question.
         - Thus each column (and each row separately) has a numerical value that can be used to either partially or totally distinguish them , and, which can be used to uniquely number the nodes.
         - These sorted values provide an entropy spectral curve for the column and also the row entropy distribution that can be used to profile the topology and to compare two topologies to see if they are different and if so where.

(a) This removes the very difficult problem of the n! versions of a network being impossible to compare.
(b) This also allows us to compare a topology at one time to a later time to see where (i.e. at which specific nodes) it has changed and by how much.

6) The Algorithm for Network Analysis with Entropy Metrics:

a) It is easier here to bypass other details of the technical foundation and give the exact prescription from the following algorithmic steps:

   i) (A1) Set the diagonal terms C_{jj} of a connection matrix to be equal to the negative of the sum all elements in that respective column ($C_{jj} = - \Sigma_\iota C_{ij}$) (and then later redo all this for the rows instead of the columns). Then divide every element of the matrix by the negative of the trace *n thus 'normalizing' the matrix to have a trace of '-n'. It can be shown that this matrix is the infinitesimal generator of a continuous Markov transformation.

   ii) (A2) Compute the associated Markov matrix as $M(s) = e^{sC}$ using any number of terms and one will always get, in any order, a Markov matrix where all matrix elements are non-negative and the sum of all elements in any column is '1'. The number of expansion terms used represents the degrees of separation thus incorporated.

   iii) (A3) Since each column has only non-negative elements and each sums to unity, it now follows that these elements can be interpreted as probabilities and thus support a definition of entropy (such as the Renyi' entropy $S_j = - \log \Sigma_i (M_{ij})^2$).

   iv) (A4) As this S_j is defined for each column (node) and as there is no natural order for the nodes, we may sort the nodes in order of these values. For real values there is rarely a degeneracy, but if there are two or more equal values, then a similar procedure, when performed on the rows, will usually distinguish the order. In any case these sorted values present an 'entropy' spectra for the columns which can be plotted as a curve, and likewise one obtains another entropy spectral curve for the rows. These two curves for the row and column entropy spectra are specific to the network topology, represent the incoming and outgoing order/disorder of connectivity. And must be identical if the topologies are identical thus allowing topological comparisons without the n! combinatorial problems. Furthermore, the 'distance between these entropy spectral curves' is indicative of the 'distance between the two topologies'.

   v) (A5) The M matrix now serves as a model for any network as a combination of conserved flows among the components of a vector upon which it acts. Specifically the eigenvectors become those linear combinations of nodes that have unique associated eigenvalues representing the rate of flow toward equilibrium, or when complex, the angular frequency of flow cycles in the approach to equilibrium.

b) Consequences for the listed problems:

   i) This analysis solves problems P2 (intuitive eigenvalue..) and P3 (well defined mathematical foundation).

   ii) Our analysis also solves problem P1 (node numbering and comparison of two topologies using entropy spectra) for essentially all practical cases.

   iii) The combination of eigenvalue analysis (for small networks) and especially entropy spectra also now provide valuable metrics for identifying aberrant dynamical changes in networks as we have previously shown thus addressing problem P7 with entropy spectra and eigenvalues.

7) **Database of Attributes for Nodes and Links**

a) One of the most challenging aspects of networks is that they can be of astronomical size since when there at n nodes, there are n^2 elements in the connection matrix and n! different methods of representing the same network.
   i) For even moderate size networks, this can overwhelm any computational system. We propose using a technique for collapsing many nodes into a single node based upon attributes in a database associated with each node (and also each link).

b) We will maintain a database of attributes (fields) associated with each node (i.e. multiple tables each with vectors of properties such as demographic profiles of the person, entity, agency etc) as well as such tables connected to the links among the nodes.
   i) We will utilize this node attribute vector to take very large networks and collapse them based upon values of these attributes.
      - A very simple example would be to consider the network of all persons in SC which would be a 4M * 4M array of linkages.
      - By collapsing nodes into groups representing decade age groups (0-9, 10-19, ..) we obtain a network of only 10 nodes thereby eliminating extensive noise and fluctuations!
      - Not only can that network be studied but each decade age group in itself can be studied treating the remaining nodes as the ‘exterior node’.

c) This technique has been essential in the physical sciences where we can look at a baseball as a single mass and then to study its dynamics without considering the internal intermolecular, atomic, or nuclear forces.
   i) Then we study those internal network aspects separately.
   ii) Thus a collapsed network can have important eigenvalue, entropy or other topological metric characteristics that can be uncovered but which would be otherwise difficult to identify in a full network of millions of nodes.
   iii) This will range from the collapsible BEA encoding of IO sectors, to the personal demographic profiles of CDC health statistics data, and to other attributes that will be maintained for dynamical network evolution generation described below, and certainly for user’s submitted networks.
   iv) We intend to automate this process to allow for a network to undergo an automatic reduction by summing the connections of those nodes that have the same attribute connecting to another node with a differing attribute value.
      - Likewise the system will output the internal connectivity of a collapsed node.
      - These two algorithms can thus separately measure the internal (sub-net) topology and the external topology (among the collapsed sub-nets).
      - We shall seek invariant metrics for diverse methods of collapsing these dynamical networks and shall seek topological similarities among the reduced systems thus allowing for network classification methods.

8) A New Model for Self Evolving Networks

a) In order to study network dynamics we propose to build a self-evolving network simulation environment that utilizes simple rules within a framework in order to evolve networks thus allowing the study of how dynamical evolution of networks is tied to rule structure.

i) That even simple rules can spawn very complex behavior has been explored in the Game of Life systems and in Steven Wolfram's "A New Kind of Science" although our environment will be much richer and different.

b) A model which we here suggest is that every social node 'seeks to expand its connectivity', in order to better survive, by having a set of 'needs' represented by a vector of properties for node 'i' as w_i^α that are sought from other nodes and a set of abilities v_i^α that are offered to other nodes.

i) After creation, a node is bonded to 'parent nodes' only and after periods of time becomes sequentially bonded to the closest of the nodes that are linked to the parents.

ii) In our proposed model, a given node will form the scalar product of their need vector with the ability vector of a linked node giving the $\cos(\theta_{ij}) = \Sigma_\alpha w_i^\alpha v_j^\alpha$ thus giving a new value of C_{ij} (as the square of that cos value to guarantee non-negativity) but only for those nodes to which a node has been previously linked.

iii) This angle will measure the 'proximity' of 'need' to 'ability'.

iv) The properties can be named (food, water, friendship…) but that is unnecessary as we just want to explore the results of the formal evolution of the network.

v) We shall also 'shape' the need and ability vectors for nodes over time to reflect a cycle from birth to aging and death as well as to give semi-random profiles to each vector's evolving profile and thereby allowing the connectivity matrix to evolve by both creating new nodes and allowing older nodes to be extinguished.

vi) We will utilize information from the national social health network that we construct above to inform suggested profiles of these vectors over time in a realistic way. We suggest that this is a reasonable model for how nodes seek to enhance some links and diminish other link connections.

c) The self-evolving dynamical network will allow us to see what types of networks, topologies, structures, clusters and other features can evolve from different sets of rules.

i) We will be seeking what types of rules give stable structures and how they evolve as well as how similar these systems are to the networks described elsewhere.

ii) More precisely, how do different rules, for changing the connectivity and for the creation and dissolving of new nodes, relate to differing network structures.

iii) This will provide a totally new environment for network evolution and allow us to study network dynamics from a rule based system rather than to attempt to model the highly nonlinear aspects of network evolution.

iv) The potential problems center around how to best model the attribute and need vectors over time and as a distribution over fields.

v) We will also be consulting with a number of psychologists, sociologists, economists, and social network experts to get advice in using the system to experiment with network dynamics and different rule sets.

vi) Although we do not foresee difficulties in the construction of the system as described and envisioned, there will be anticipated problems in making this self evolving system give similar results to existing network topologies.

- Which rule sets will give rise to what kind of structures will be very complex inquiry.
- We will seek those dynamical evolutions that do not kill off all the nodes, have regeneration that is not exponentially overpowering, and which at the same time does not just lock up on an uninteresting configuration or topology such as a clique.